[9]Degani, D., "Numerical Study of the Effect of Geometrical Disturbances on Vortex Asymmetry," *AIAA Journal*, Vol. 29, No. 4, 1991, pp. 560–566.

[10]Degani, D., "Instability of Flows over Bodies at Large Incidence," *AIAA Journal*, Vol. 30, No. 1, 1992, pp. 94–100.

J. C. Hermanson
Associate Editor

In-Plane Vibrations of Nonuniform Circular Beams

Sen Yung Lee* and Jeng Chung Chao†
National Cheng-Kung University, Tainan 701, Taiwan, Republic of China

Nomenclature

$A(s)$	=	cross-sectional area of the beam
$E(s)$	=	Young's modulus of the material
F_{rv}	=	force per unit arc length in the r direction caused by the flexural deflection parameter V
$F_{\theta v}$	=	force per unit arc length in the θ direction caused by the flexural deflection parameter V
$I_z(s)$	=	area moment of inertia of the beam section about the z axis
L	=	total length of the neutral axis of the beam
R	=	radius of the curved beam
s	=	arc length variable, $s = R\theta$
$U(s)$	=	longitudinal displacement parameter
u	=	neutral axis displacement of the beam in the θ direction
$V(s)$	=	flexural displacement parameter
v	=	neutral axis displacement of the beam in the r direction
β	=	angular frequency of the in-plane vibration
$\rho(s)$	=	mass per unit volume of the beam

Introduction

CURVED-BEAM structures have been used in many aerospace, mechanical, and civil engineering applications such as curved wires in missile-guidance floated gyroscopes, stiffeners in aircraft structures, turbomachinery blades, curved girder bridges, brake shoes within drum brakes, spring design, and tire dynamics. It can also be used as a simplified model of a shell structure. Research in this area can be traced back to the 19th century.[1,2] An interesting review can be found in the review papers by Markus and Nanasi,[3] Laura and Maurizi,[4] Childamparam and Leissa,[5] and Auciello and De Rosa.[6]

In general, the out-of-plane and the in-plane vibrations of a general plane curved beam are coupled. However, if the cross section of the curved beam is doubly symmetric and the thickness of the beam is small in comparison with the radius of the beam, then the out-of-plane and the in-plane vibrations are independent.[2,7]

Many investigators have studied the in-plane vibrations of curved beams. The associated governing differential equations are two coupled differential equations in the flexural and the longitudinal displacements. If the beam is uniform, then the coefficients of the differential equations are constants. After some simple arithmetic operations the two coupled differential equations can be reduced into a sixth-order ordinary differential equation with constant coefficients in the flexural displacement.[2] Hence the problem can be solved by different analytical methods, and the exact solutions can be obtained.[2,8–10] However, it is not the case for the nonuniform beams.

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*Professor, Mechanical Engineering Department; sylee@mail.ncku.edu.tw.

†Graduate Student, Mechanical Engineering Department.

Because of the complexity in the coefficients of the governing differential equations, the two-coupled differential equations have never been uncoupled and reduced into sixth-order ordinary differential equations. Exact solutions for the curved nonuniform beam problem are only found in the work done by Suzuki and Takahashi,[11] who gave an exact series solution to the beams with the same boundary conditions at both ends. Nevertheless, their method has difficulty in handing the problems with other kind of boundary conditions. Hence, the problems were studied mainly by approximated methods such as the Rayleigh–Ritz method,[12] the Galerkin method,[13] the transfer matrix method,[14] the discrete Green function method,[15] and the asymptotic analysis of the equations of free vibrations.[16]

In this Note, by introducing two physical parameters, the analysis is simplified, and the explicit relations between the flexural displacement and the longitudinal displacement of the system are derived. With these explicit relations the coupled governing characteristic differential equations can be uncoupled and reduced to sixth-order ordinary differential equations with variable coefficients in the flexural displacement. In addition, by employing the explicit relations, one only has to measure one variable instead of two simultaneously while performing experimental studies of curved beams. Hence, it greatly reduces the difficulty in experimental measurements.

When the radius of a curved beam becomes infinite, the curved beams reduces to a straight beam. Consequently, the sixth-order ordinary differential equation in the flexural displacement should reduce to a fourth-order ordinary differential equation. However, it is not possible to perform this limiting process from the reduced sixth-order ordinary differential equation for the beam,[2] and the limiting study had never been successfully explored before. In this Note, by employing the explicit relations, a successful limiting study is revealed.

Coupled Governing Differential Equations

Consider the in-plane vibrations of nonuniform curved beams of constant radius R and doubly symmetric cross section, as shown in Fig. 1. If the thickness of the beam is small in comparison with the radius of the beam, without considering the shear deformation, the rotary inertia, and the warping effects the governing differential equations for the in-plane vibrations are two coupled differential equations in terms of the flexural and the longitudinal displacements.[7]

For time-harmonic in-plane vibrations of curved beams with angular frequency β, one assumes

$$u(s, t) = U(s)e^{i\beta t} \tag{1}$$

$$v(s, t) = V(s)e^{i\beta t} \tag{2}$$

The two coupled governing characteristic differential equations of the system are

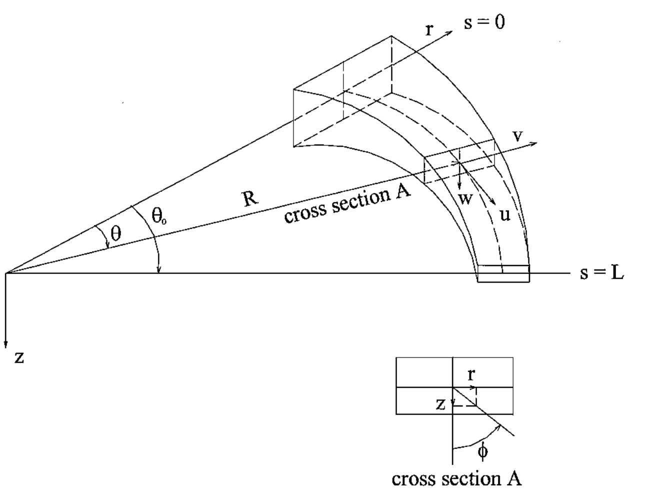

Fig. 1 Geometry and coordinate system of a curved nonuniform beam of constant radius.

$$\{EI_z[(1/R^2)U' - (1/R)V'']\}' + \{EA[U' + (1/R)V]\}' + \rho A\beta^2 U = 0 \tag{3}$$

$$\{EI_z[(1/R)U' - V'']\}'' - EA[(1/R)U' + (1/R^2)V] + \rho A\beta^2 V = 0 \tag{4}$$

where the primes denotes differentiation with respect to the s variable. The associated boundary conditions are, at $s = 0$ and L,

$$EI_z[(1/R^2)U' - (1/R)V''] + EA[U' + (1/R)V] = 0 \quad \text{or} \quad U = 0 \tag{5}$$

$$\{EI_z[(1/R)U' - V'']\}' = 0 \quad \text{or} \quad V = 0 \tag{6}$$

$$EI_z[(1/R)U' - V''] = 0 \quad \text{or} \quad V' = 0 \tag{7}$$

Uncoupled Governing Differential Equation in Terms of the Flexural Displacement Parameter V

Nonuniform Curved Beams

To simplify the analysis, one defines the following two physical parameters

$$F_{\rm rv} = (EI_z V'')'' + EA(V/R^2) - \rho A\beta^2 V \tag{8}$$

$$F_{\theta v} = [EI_z(V''/R)]' - [EA(V/R)]' \tag{9}$$

In terms of $F_{\rm rv}$ and $F_{\theta v}$, the two coupled governing charcateristic differential equations of the system Eqs. (3) and (4) can be rewritten as

$$\{EI_z[(1/R^2)U']\}' + (EAU')' - F_{\theta v} + \rho A\beta^2 U = 0 \tag{10}$$

$$\{EI_z[(1/R)U']\}'' - EA[(1/R)U'] - F_{\rm rv} = 0 \tag{11}$$

Substituting Eq. (10) and the equation resulting from differentiating Eq. (10) into Eq. (11), one obtains

$$\frac{1}{R}U' = \frac{a_1}{R}U + \frac{1}{g_1}\left\{\frac{1}{EI_z}\left[\frac{2}{R}\left(\frac{EI_z}{p_v}\right)' F_{\theta v} - F_{\rm rv}\right] + \frac{F'_{\theta v}}{p_v R}\right\} \tag{12}$$

where p_v, g_1, and a_1 are given in the Appendix. Differentiating Eq. (12) and combining it with Eqs. (10) and (12), one can explicitly express U, U', and U'' in terms of the flexural displacement parameter V

$$\frac{1}{R}U = \frac{1}{a_2}q_v + \frac{1}{g_1 a_2}\left[\frac{a_3}{EI_z}F_{\rm rv} - \frac{a_3}{p_v R}F'_{\theta v} + \left(\frac{g_1}{p_v} - a_3 a_5\right)\frac{F_{\theta v}}{R}\right] \tag{13}$$

$$\frac{1}{R}U' = \frac{a_1}{a_2}q_v + \frac{1}{g_1 a_2}\left[\frac{a_4}{EI_z}F_{\rm rv} - \frac{a_4}{p_v R}F'_{\theta v} + \left(\frac{a_1 g_1}{p_v} - a_4 a_5\right)\frac{F_{\theta v}}{R}\right] \tag{14}$$

$$\frac{1}{R}U'' = \left(\frac{\rho A\beta^2 + a_1 p'_v}{a_2 p_v}\right)q_v - \left(\frac{a_3\rho A\beta^2 + a_4 p'_v}{p_v g_1 a_2}\right) \times \left(\frac{F_{\rm rv}}{EI_z} - \frac{F'_{\theta v}}{p_v R}\right) + \frac{1}{p_v a_2 R}\cdot\left[\frac{1}{p_v}\left(\rho A\beta^2 + a_1 p'_v\right) - \frac{a_3}{g_2}\left(\rho A\beta^2 a_1 + a_2 p'_v\right)\right]F_{\theta v} \tag{15}$$

where

$$q_v = \left[\frac{1}{g_1}\left(\frac{F_{\rm rv}}{EI_z} - \frac{F'_{\theta v}}{p_v R} - \frac{a_5}{R}F_{\theta v}\right)\right]' \tag{16}$$

and g_2 and a_2–a_5 are given in the Appendix. Substituting Eqs. (14) and (15) into Eq. (11), one obtains the uncouple governing differential equation, which is a sixth-order ordinary differential equation in terms of the flexural displacement parameter V:

$$\begin{aligned}
&\Bigg(EI_z\Bigg\{\left(\frac{\rho A\beta^2 + a_1 p'}{a_2 p_v}\right)\left[\frac{1}{g_1}\left(\frac{F_{\rm rv}}{EI_z} - \frac{F'_{\theta v}}{p_v R} - \frac{a_5}{R}F_{\theta v}\right)\right]' \\
&- \left(\frac{a_3\rho A\beta^2 + a_4 p'_v}{p_v g_1 a_2}\right)\left(\frac{F_{\rm rv}}{EI_z} - \frac{F'_{\theta v}}{p_v R}\right) + \frac{1}{p_v a_2 R} \\
&\times\left[\frac{1}{p_v}\left(\rho A\beta^2 + a_1 p'_v\right) - \frac{a_3}{g_2}\left(\rho A\beta^2 a_1 + a_2 p'_v\right)\right]F_{\theta v}\Bigg\} \\
&+ (EI_z)'\left\{\frac{a_1}{a_2}\left[\frac{1}{g_1}\left(\frac{F_{\rm rv}}{EI_z} - \frac{F'_{\theta v}}{p_v R} - \frac{a_5}{R}F_{\theta v}\right)\right]'\right. \\
&\left.+ \frac{1}{g_1 a_2}\left[\frac{a_4}{EI_z}F_{\rm rv} - \frac{a_4}{p_v R}F'_{\theta v} + \left(\frac{a_1 g_1}{p_v} - a_4 a_5\right)\frac{F_{\theta v}}{R}\right]\right\}\Bigg)' \\
&- EA\left\{\frac{a_1}{a_2}\left[\frac{1}{g_1}\left(\frac{F_{\rm rv}}{EI_z} - \frac{F'_{\theta v}}{p_v R} - \frac{a_5}{R}F_{\theta v}\right)\right]' + \frac{1}{g_1 a_2}\left[\frac{a_4}{EI_z}F_{\rm rv}\right.\right. \\
&\left.\left. - \frac{a_4}{p_v R}F'_{\theta v} + \left(\frac{a_1 g_1}{p_v} - a_4 a_5\right)\frac{F_{\theta v}}{R}\right]\right\} - F_{\rm rv} = 0
\end{aligned} \tag{17}$$

The associated boundary conditions in terms of the flexural displacement parameter V can be obtained by substituting the explicit relations (15–17) into the boundary conditions (5–7).

Uniform Circular Beams

For uniform circular beams

$$a_1 = a_3 = a_5 = 0, \qquad a_2 = -a_4 = \rho A\beta^2\big/p_v$$
$$g_1 = A/I_z + (\rho A/p_v)\beta^2 \tag{18}$$

$F_{\rm rv}$ and $F_{\theta v}$ now are

$$F_{\rm rv} = EI_z V'''' + EA(V/R^2) - \rho A\beta^2 V \tag{19}$$

$$F_{\theta v} = EI_z(V'''/R) - EA(V'/R) \tag{20}$$

Substituting Eqs. (18–20) into Eq. (17), it becomes a sixth-order differential equation with constant coefficients:

$$V^{(6)} + \left(\frac{2}{R^2} + \frac{\rho\beta^2}{E}\right)V'''' + \left[\frac{1}{R^4} - \frac{\rho}{E}\beta^2\left(\frac{A}{I_z} + \frac{1}{R^2}\right)\right]V'' + \frac{\rho A}{EI_z}\beta^2\left(\frac{1}{R^2} - \frac{\rho\beta^2}{E}\right)V = 0 \tag{21}$$

where $V^{(6)}$ indicates the sixth derivate of V with respect to s. This equation is the uncoupled governing differential equation of an uniform circular beam and is the same as the one given by Lee.[17]

Nonuniform Straight Beams

When the radius of a curved beam approaches infinite, the curved beam becomes a straight beam. Hence, by setting R as infinite, the reduced sixth-order ordinary differential equations (17) and (21) should reduce to fourth-order differential equations. However, it can be found that it is not possible to take such a direct limiting study. In this section, by employing the explicit relations (14) and (15), a successful limiting study is revealed in the following.

By letting R be infinite in Eqs. (10) and (11), F_{rv} and $F_{\theta v}$ reduce to

$$F_{\theta v} = 0 \tag{22}$$

$$F_{rv} = (EI_z V'')'' - \rho A \beta^2 V \tag{23}$$

Equation (17) becomes

$$\left\{ EI_z \left[\left(\frac{\rho A \beta^2 + a_1 p_v'}{a_2 p_v} \right) \left(\frac{F_{rv}}{g_1 EI_z} \right)' - \left(\frac{a_3 \rho A \beta^2 + a_4 p_v'}{p_v g_1 a_2 EI_z} \right) F_{rv} \right] + (EI_z)' \left[\frac{a_1}{a_2} \left(\frac{F_{rv}}{g_1 EI_z} \right)' + \frac{a_4}{g_1 a_2 EI_z} F_{rv} \right] \right\}' - EA \left[\frac{a_1}{a_2} \left(\frac{F_{rv}}{g_1 EI_z} \right)' + \frac{a_4}{g_1 a_2 EI_z} F_{rv} \right] - F_{rv} = 0 \tag{24}$$

The explicit relations Eqs. (14) and (15) are reduced to

$$0 = \frac{a_1}{a_2} \left(\frac{F_{rv}}{g_1 EI_z} \right)' + \frac{a_4}{g_1 a_2 EI_z} F_{rv} \tag{25}$$

$$0 = \left(\frac{\rho A \beta^2 + a_1 p_v'}{a_2 p_v} \right) \cdot \left(\frac{F_{rv}}{g_1 EI_z} \right)' - \left(\frac{a_3 \rho A \beta^2 + a_4 p_v'}{p_v g_1 a_2 EI_z} \right) F_{rv} \tag{26}$$

Substituting Eqs. (25) and (26) to Eq. (24), the sixth-order governing differential equation can be reduced to a fourth-order ordinary differential equation

$$F_{rv} = (EI_z V'')'' - \rho A \beta^2 V = 0 \tag{27}$$

By setting R as infinite, the boundary conditions (6) and (7) become

$$(EI_z V'')' = 0 \qquad \text{or} \qquad V = 0 \tag{28}$$

$$V'' = 0 \qquad \text{or} \qquad V' = 0 \tag{29}$$

Equation (27) and Eqs. (28) and (29) are the governing differential equation and the associated boundary conditions for the flexural vibration of a straight nonuniform beam, with which[18] we are familiar.

Exact Fundamental Solutions and Frequency Equation

The uncoupled governing differential equations (22) and (47) for the in-plane vibration of curved nonuniform beams can be expressed as sixth-order differential equations with variable coefficients. It has been shown that if the material and geometric properties of curved beams are in arbitrary polynomial forms then the six exact, fundamental solutions of the governing characteristic differential equation can be obtained.[7] After substituting these fundamental solutions into the associated boundary conditions, the frequency equation and natural frequencies of the beams can be obtained, consequently.

Conclusions

In this Note, by introducing two physical parameters, the analysis of the in-plane vibrations of nonuniform curved beams with constant radius is simplified. The explicit relations between the in-plane flexural and the longitudinal displacements of the beam are derived. With these explicit relations one only has to measure one variable instead of two simultaneously while performing experimental studies of curved beams. Hence, it greatly reduces the difficulty in experimental measurements. In addition, the two coupled governing differential equations are reduced to one complete sixth-order ordinary differential equations with variable coefficients in the in-plane flexural displacement. Consequently, if the material and the geometric properties of the beam are in arbitrary polynomial forms then the exact vibration solutions of the system can be obtained. Finally, a limiting study from the curved beam theory to the straight beam theory is successfully revealed.

Appendix: Parameters

The forms of p_v, p, g_1–g_2, and a_1–a_5 are

$$p_v = EA\left[1 + (\gamma_z / R)^2\right]$$

$$p = \left\{ \frac{EI_z}{(EA)^3} \left[\rho A \beta^2 - \frac{EA}{R^2} - (EA)'' \right] \right\}'$$

where

$$\gamma_z = \sqrt{\frac{I_z}{A}}$$

$$g_1 = -\frac{p_v}{EI_z} \left(\frac{EI_z}{p_v} \right)'' + \frac{A}{I_z} + \frac{\rho A}{p_v} \beta^2$$

$$g_2 = 3 \left[\frac{(EA)'}{(EA)^2} \right]' EA + \frac{A}{I_z} + 2 \frac{(EA)'(EI_z)'}{(EA)(EI_z)} - \frac{\rho \beta^2}{E} + \frac{1}{R^2}$$

$$a_1 = -\frac{p_v}{g_1} \left(\frac{\beta}{EI_z} \right)^2 \left[\rho A \left(\frac{EI_z}{p_v} \right)^2 \right]'$$

$$a_2 = -\frac{1}{g_1} \left(\frac{\beta}{EI_z} \right)^2 \left\{ p_v \left[\rho A \left(\frac{EI_z}{p_v} \right)^2 \right]' \right\}' + \frac{\beta}{g_1 R E I_z} \left(\frac{1}{R^2} + \frac{1}{\gamma_z^2} \right) \times \left[\rho A \left(\frac{EI_z}{p_v} \right)^2 \right]' \left[\left(\rho A \beta^2 + p_v g_1 \right) \left(\frac{EI_z}{p_v} \right)^2 \right]' + \frac{\rho A \beta^2}{p_v}$$

$$a_3 = -\frac{\beta^2}{g_1 EI_z} \left(\frac{1}{R^2} + \frac{1}{\gamma_z^2} \right) \left[\rho A \left(\frac{EI_z}{p_v} \right)^2 \right]' + \frac{p_v'}{p_v}$$

$$a_4 = \left\{ \frac{\beta^2}{g_1 EI_z} \left(\frac{1}{R^2} + \frac{1}{\gamma_z^2} \right) \left[\rho A \left(\frac{EI_z}{p_v} \right)^2 \right]' \right\}' - \frac{\rho A \beta^2}{p_v}$$

$$a_5 = \frac{2}{EI_z} \left(\frac{EI_z}{p_v} \right)'$$

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References

[1] Michell, J. H., "The Small Deformation of Curves and Surfaces with Application to the Vibrations of a Helix and a Circular Ring," *Messenger of Mathematics*, Vol. 19, 1890, pp. 68–76.

[2] Love, A. E. H., *A Treatise on the Mathematical Theory of Elasticity*, 4th ed., Dover, New York, 1944, pp. 444–454.

[3] Markus, S., and Nanasi, T., "Vibrations of Curved Beams," *The Shock and Vibration Digest*, Vol. 13, No. 4, 1981, pp. 3–14.

[4] Laura, P. A. A., and Maurizi, M. J., "Recent Research on Vibrations of Arch-Type Structures," *Shock and Vibration Digest*, Vol. 19, No. 1, 1987, pp. 6–9.

[5] Childamparam, P., and Leissa, A. W., "Vibrations of Planar Curved Beams, Rings and Arches," *Applied Mechanics Reviews*, Vol. 46, No. 9, 1993, pp. 467–483.

[6] Auciello, N. M., and De Rosa, M. A., "Free Vibrations of Circular Arches: A Review," *Journal of Sound and Vibration*, Vol. 176, No. 4, 1994, pp. 433–458.

[7] Lee, S. Y., and Chao, J. C., "Exact Out-of-Plane Vibrations for Curved Nonuniform Beams," *Journal of Applied Mechanics* (to be published).

[8] Den Hartog, J. P., "The Lowest Natural Frequency of Circular Arcs," *Philosophical Magazine*, Vol. 5, Series 7, 1928, pp. 400–408.

[9] Morley, L. S. D., "The Flexural Vibrations of a Cut Thin Ring," *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 11, Pt. 4, 1958, pp. 491–497.

[10]Mau, S. T., and Williams, A. N., "Green's Function Solution for Arch Vibration," *Journal of Engineering Mechanics*, Vol. 114, No. 7, 1988, pp. 1259–1264.

[11]Suzuki, K., and Takahashi, S., "Inplane Vibrations of Curved Bars with Varying Cross-Section," *Bulletin of the Japan Society of Mechanical Engineers*, Vol. 25, No. 205, 1982, pp. 1100–1107.

[12]Laura, P. A. A., Bambill, E., Filipich, C. P., and Rossi, R. E., "A Note on Free Flexural Vibrations of a Non-Uniform Elliptical Ring in its Plane," *Journal of Sound and Vibration*, Vol. 126, No. 2, 1988, pp. 249–254.

[13]Lecoanet, H., and Piranda, J., "Inplane Vibrations of Circular Rings with a Radically Variable Thickness," *Journal of Vibration Acoustics Stress Reliability Design*, Vol. 105, No. 1, 1983, pp. 137–143.

[14]Murthy, V. R., and Nigam, N. C., "Dynamic Characteristics of Stiffened Rings by Transfer Matrix Approach," *Journal of Sound and Vibration*, Vol. 39, No. 2, 1975, pp. 237–245.

[15]Kawakami, M., Sakiyama, T., Matsuda, H., and Morita, C., "In-Plane and Out-of-Plane Free Vibrations of Curved Beams with Variable Sections," *Journal of Sound and Vibration*, Vol. 187, No. 3, 1995, pp. 381–401.

[16]Tarnopolskaya, T., De Hoog, F., Fletcher, N. H., and Thwaites, S., "Asymptotic Analysis of the Free In-Plane Vibrations of Beams with Arbitrarily Varying Curvature and Cross-Section," *Journal of Sound and Vibration*, Vol. 196, No. 5, 1996, pp. 659–680.

[17]Lee, L. S. S., "Vibrations of an Intermediately Supported U-Bend Tube," *Journal of Engineering for Industry*, Vol. 97, No. 1, 1975, pp. 23–32.

[18]Meirovitch, L., *Analytical Methods in Vibrations*, Macmillan, New York, 1967.

A. Berman
Associate Editor

Postbuckling of Thermally Stressed Composite Plates

Lazarus Teneketzis Tenek*
Aristotle University of Thessaloniki,
540 06 Thessaloniki, Greece

I. Introduction

STRUCTURES made of composite materials often operate at elevated temperatures. At these temperatures thermomechanical stressing may occur. Composite structures that are thermally stressed may buckle and postbuckle from subsequent application of elastic loads. It is, therefore, of interest to examine the postbuckling behavior of structures made of composite materials that are thermally stressed. Of particular interest is the effect of initial thermal stressing to the subsequent postbuckling behavior of composite panels. The present study considers two laminated composite plates, namely, an eight-layer $(0/90/0/90)_s$ cross-ply laminate and a eight-layer quasi-isotropic $(45/-45/0/90)_s$ composite plate. Both plates are thermally stressed via the application of various temperatures. Following thermal stressing an axial load is incrementally applied, and the postbuckling behavior is examined.

Some analytical and numerical studies concerning the behavior of composite structures at elevated temperatures have been reported.[1–5] Most of these studies, however, are concerned solely with thermal buckling and postbuckling of composite plates. The present study considers the laminated plates to be in a state of thermal stressing before application of the elastic compressive loads. The effect of initial thermal stressing to the final postbuckling behavior is assessed.

II. Computational Experiments

Figure 1 shows a laminated composite plate along with all geometrical and material properties. The left edge of the plate cannot move in the three directions, whereas for all other edges vertical

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*Research Scientist, Laboratory of Mechanics and Materials, Box 468.

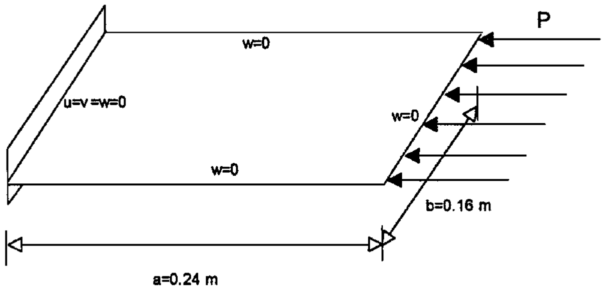

Fig. 1 Laminated composite plate; geometrical and material data: $E_1 = 150$ GPa, $E_2 = E_3 = 10$ GPa, $G_{12} = G_{13} = 6$ GPa, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$, $a_{t1} = 2.5 \times 10^{-8}$ °C^{-1}, $a_{t2} = 30 \times 10^{-6}$ °C^{-1}, and $h_L = 3.125 \times 10^{-4}$ m.

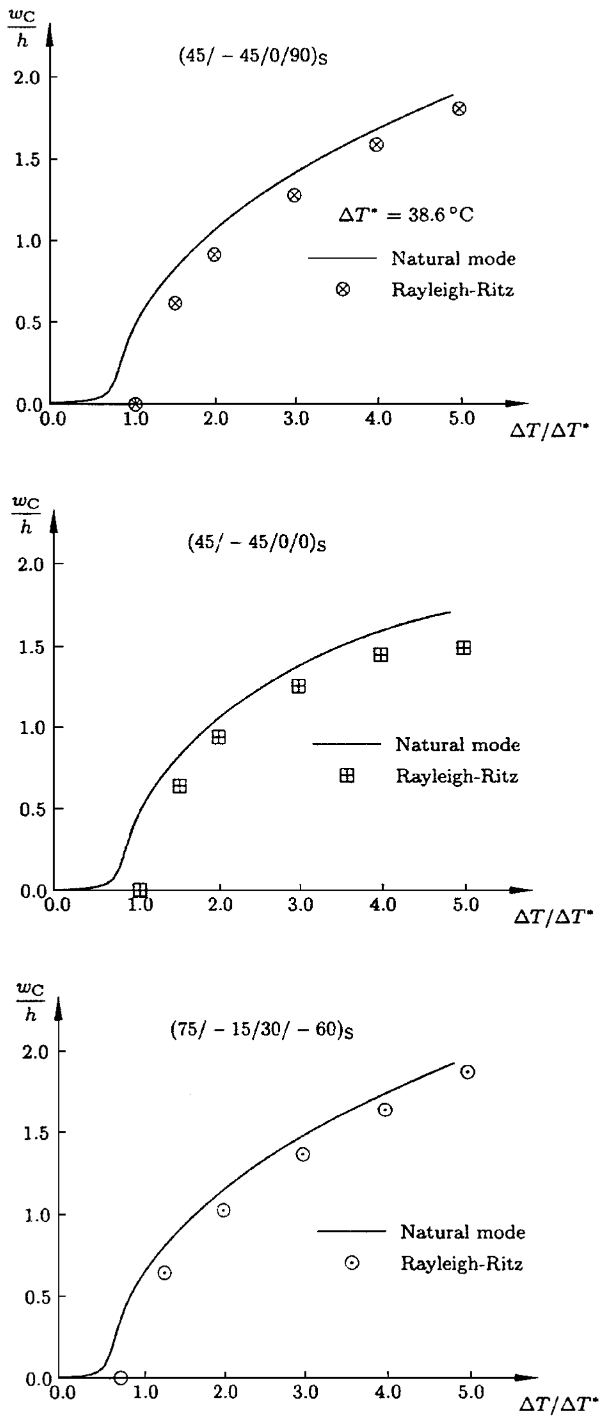

Fig. 2 Comparison of present natural-mode method with reported results [Rayleigh–Ritz method (Ref. 4)] for thermal postbuckling of three composite plates.